An Optimal Robust Equidistribution Method for 2D Grid Generation Based on Monge-Kantorovich Optimization

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Equidistribution of a positive monitor function (a density) is a central concept in grid adaptation. An example of its utility is in equidistribution of truncation errors in a discretization scheme, leading to minimization of the total error.

We have developed a new equidistribution method for 2D grid adaptation, based on Monge-Kantorovich optimization. The method is based on a rigorous variational principle, in which the L_2 norm of the grid displacement is minimized, constrained *locally* to produce a prescribed positive cell volume distribution. This procedure minimizes the grid velocity in a time-stepping context, and avoids the central problem of Lagrangian methods: grid tangling. Our method is in contrast with commonly used variational grid adaptation schemes, in which a linear combination of cost integrals measuring different grid properties is used. In these methods, the scheme is a compromise between these different measures, and neither prevention of grid tangling nor exact equidistribution is achieved.

Our method involves finding a new grid x'(ξ),(ξ is a logical rectangular grid) with density ρ'(x'), in terms of the old grid x with density ρ(x). Optimization leads to x'= x' + $\nabla\Phi$ with

$$\nabla^2\Phi + H[\Phi] = \frac{\rho(\mathbf{x})}{\rho'(\mathbf{x}')} - 1, \qquad (1)$$

where H is the Hessian of Φ. This is the Monge-Ampère equation, a single, nonlinear elliptic scalar equation with no free parameters and with well-known existence and uniqueness properties. Once the solution is found, the adapted grid should not fold since volumes are prescribed to be positive, and displacement of grid points is minimized. We solve this equation numerically with a Jacobian-free Newton-Krylov method. The ellipticity property of Eq. (1) allows multigrid preconditioning techniques to be used effectively. The resulting numerical method is algorithmically scalable and uses a negligible amount of computational time and storage.

We have also shown that our variational principle maximizes grid smoothness (minimizes grid distortion), as measured by the L_2 norm of the trace of the metric tensor. In Figs. 1-3, we show the resulting grid for two challenging cases of densities to be equidistributed. We plan to extend this grid generation formulation based on Monge-Kantorovich optimization to three dimensions.

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Funding Acknowledgments

- Department of Energy, Office of Science, Office of Advanced Scientific Computing Research
- Department of Energy, Office of Science, Office of Fusion Energy Sciences

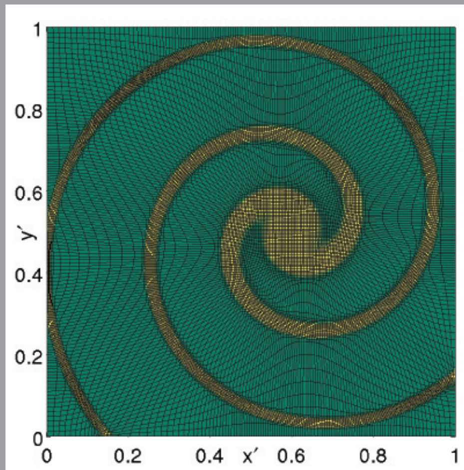

Fig. 1. The optimal grid that equidistributes a density $\rho'(x')$ (with $\rho(x) = 1$) simulating spiraling arms of vorticity in the nonlinear Kelvin-Helmholtz instability, in which Lagrangian codes can develop grid tangling.

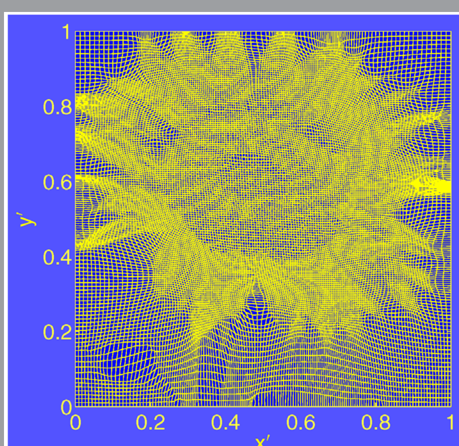

Fig. 2. Grid lines for a flower image.

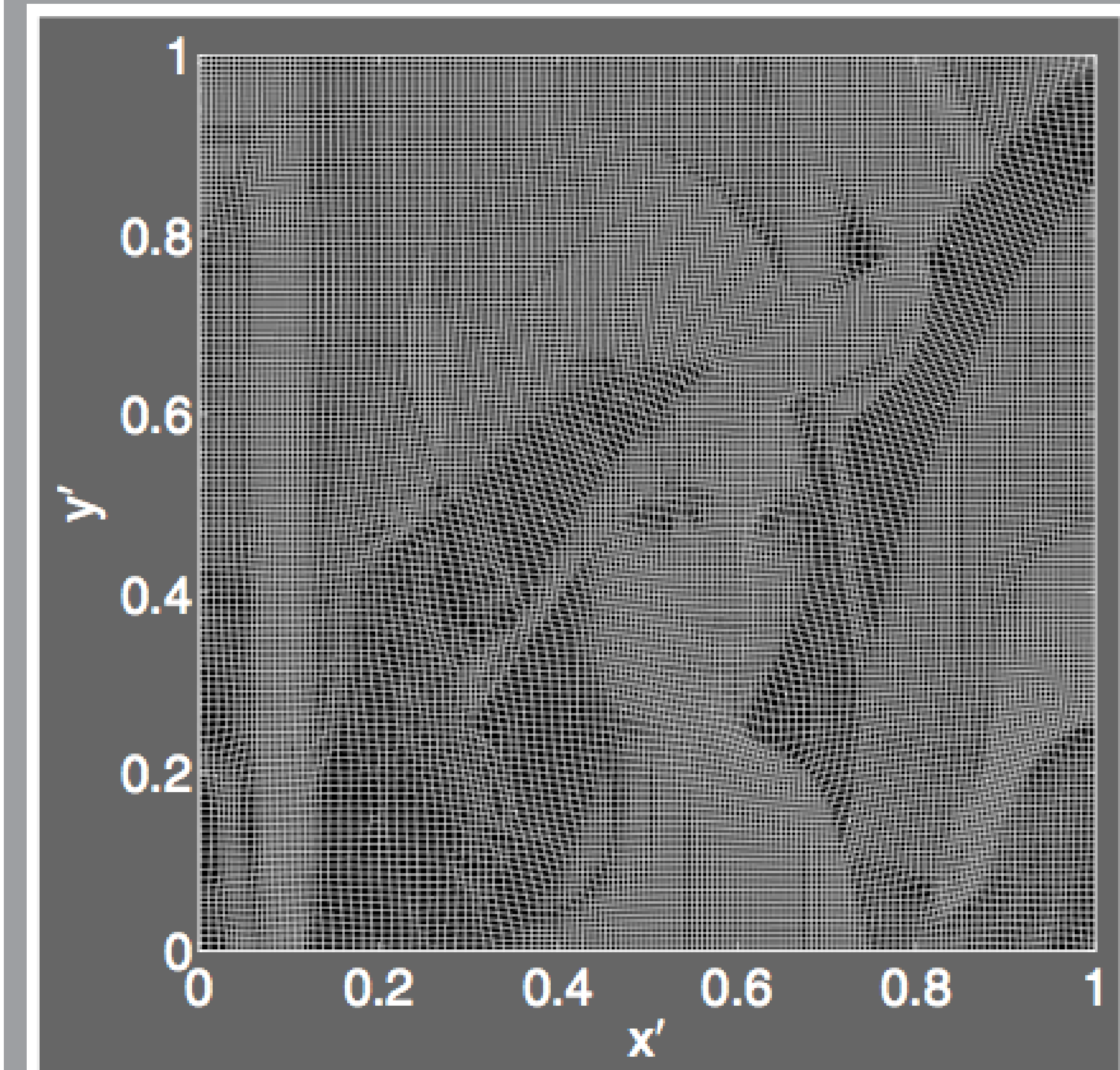

Fig. 3. Gridding up the ubiquitous Lena.